Pressure Drop Through Tapered Wide-Slit Dies. A Revised Version

Two years ago,, this journal published a paper entitled, "Pressure Drop Through Tapered Dies," and a subsequent erratum [*Appl. Polym. Sci.*, **22**, 1775 (1978) and **23**, 1587 (1979), respectively]. Since then, Dr. B. C. Sakiades of du Pont has very kindly pointed out that a factor of 2 should appear in the final equations for the pressure drop in wide-slit dies, and one of us (R.A.F.) has suggested an alternative treatment for the pressure drop in a wide-slit die with simultaneous vertical and lateral tapers.

We therefore give the following revised version:

CASE 1: WIDE SLIT OF CONSTANT WIDTH W AND A VERTICAL TAPER ANGLE θ

Elementary trigonometry shows that

$$\tan\theta = \frac{H_1 - h}{2l} = -\frac{dh}{2dl}$$

$$dh = -2dl\tan\theta,$$

$$dl = -\frac{dh}{2}\cot\theta$$

where H_1 is the entrance height and h is the height at any point l along the forward plane of flow. The exit height H_2 will appear later when taking the limits for the entire die length L.

In wide-slit dies the Rabinowitsch correction is

$$\dot{\gamma} = \frac{2n+1}{3n}\dot{\gamma}_N = \frac{2n+1}{3n}\frac{6Q}{WH^2} = \frac{Q}{wH^2}\cdot\frac{4n+2}{n}$$

In flow through a slit die without taper, the balance of forces gives

$$\tau \cdot 2L\,(W+H) = \Delta P \cdot WH$$

and in a wide slit where $W >>> H$, $W + H \simeq W$, so that $\tau = \Delta PH/2L = \eta\dot{\gamma}^n$,

$$\Delta P = \frac{2L\eta}{W^n H^{2n+1}}\left(Q\frac{4n+2}{n}\right)^n \quad \text{(A)}$$

where M and W are the total channel height and width, respectively.

We now consider the pressure drop dP for a very small length dl in a vertically tapered die and integrate with respect to h between H_1 and H_2, making the substitution for dl in terms of dh and $\cot\theta$, as above, to recover the total pressure drop ΔP over the full length L. This yields

$$\Delta P = -\eta W^{-n}\cot\theta\left(Q\frac{4n+2}{n}\right)^n \int_{H_1}^{H_2} h^{-(2n+1)}dh \quad (1)$$

and simplifying and taking the limits,

$$\Delta P = \eta\cot\theta\frac{w^{-n}}{2n}\left(Q\frac{4n+2}{n}\right)^n (H_2^{-2n} - H_1^{-2n}) \quad (2)$$

CASE 2: WIDE SLIT OF CONTRAST HEIGHT *H* AND A LATERAL TAPER ANGLE ϕ

By trignonometry, as before,

$$\tan\phi = \frac{W_1 - w}{2l} = -\frac{dw}{2dl}$$

$$dw = -2dl\tan\phi,$$

$$dl = -\frac{dw}{2}\cot\phi$$

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where W_1 is the entrance width and w is the width at any point l along the forward plane of flow. The exit width W_2 appears later when taking the limits for the entire die length L. Proceeding in precisely the same way as before, we obtain eqs. (3) and (4) corresponding to eqs. (1) and (2), respectively:

$$\Delta P = \eta H^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n \cot \phi \int_{W_1}^{W_2} W^{-n}\, dw \tag{3}$$

and

$$\Delta P = \frac{\eta}{n-1} H^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n \cot \phi (W_2{}^{1-n} - W_1{}^{1-n}) \tag{4}$$

CASE 3: INVERTED FISHTAIL DIE WITH LATERAL TAPER ANGLE ϕ AND VERTICAL TAPER ANGLE θ

As before, by trigonometry

$$h = H_1 - 2l \tan \theta$$

$$w = W_1 - 2l \tan \phi$$

But θ and ϕ are interdependent constants, and it is possible to eliminate l between these two equations; w can then be expressed as a function of h only:

$$w = W_1 - (H_1 - h) \frac{\cot \theta}{\cot \phi} \tag{5}$$

and

$$dl = -\frac{dh \cot \theta}{2} \tag{6}$$

Expressing the incremental pressure drop in the form analogous to equation (A),

$$dP = -\frac{2\, dl\, \eta}{h^{2n+1}} \left(Q \frac{4n+2}{wn}\right)^n \tag{7}$$

and substituting (5) and (6) in (7), we obtain

$$dP = +\frac{dh \cot \theta \cdot \eta}{h^{2n+1}} \left\{ Q \frac{4n+2}{n\left[W_1 - (H_1 - h)\dfrac{\cot \theta}{\cot \phi}\right]} \right\}^n \tag{8}$$

We now assume that the term $(H_1 - h)/(\cot \theta/\cot \phi)$ is small compared to W_1, that is to say, that the diminution in the die width $(W_1 - W_2)$ is small compared with W_1. Most dies conform with this. Equation (8) can be integrated to give an expression for ΔP in terms of h, θ, and ϕ. A legitimate approximation is required in order to use the binomial theorem.

We arrange terms to give

$$dP = \eta \cot \theta \left(Q \frac{4n+2}{w_1 h}\right)^n dh\, h^{-(2n+1)} \left[\frac{1}{1 - \dfrac{(H_1 - h)\cot \theta}{W_1 \cot \phi}}\right]^n$$

Using the binomial theorem on the last term,

$$dP = \eta \cot \theta \left(Q \frac{4n+2}{w_1 n}\right)^n dh\, h^{-(2n+1)} \left[1 + \frac{n(H_1 - h)}{W_1} \frac{\cot \theta}{\cot \phi}\right]$$

or

$$dP = \left[\eta \cot \theta \left(Q \frac{4n+2}{W_1 n}\right)^n\right] \left[\left(1 + \frac{nH_1}{W_1} \frac{\cot \theta}{\cot \phi}\right) h^{-(2n+1)} - \frac{n \cot \theta}{W_1 \cot \phi} h^{-2w}\right] dh \tag{9}$$

Substituting A for the first square bracket term in eq. (9) and integrating between the limits of H_1 and H_2, we now obtain the total pressure drop ΔP:

$$\Delta P = -A \int_{H_1}^{H_2} \left[\left(1 + \frac{nH_1 \cot\theta}{W_1 \cot\theta}\right) h^{-(2n+1)} - \frac{n \cot\theta}{w_1 \cot\phi} h^{-2n}\right] dh \tag{10}$$

which ultimately leads to

$$\Delta P = +A \left[\frac{(H_2{}^{-2n} - H_1)^{-2n}}{2n}\left(1 + \frac{nH_1 \cot\theta}{W_1 \cot\phi}\right) + (H_2{}^{1-2n} - H_1{}^{1-2n}) \frac{n}{W_1(1-2n)} \frac{\cot\theta}{\cot\phi}\right] \tag{11}$$

This equation is considerably simplified if we specify that the *shape* of the rectangular wide-slit cross section does not alter, i.e., that the ratio h/w is to remain constant.

Substituting (6) in (7),

$$dP = + \frac{\eta\, dh \cot\theta}{h^{2n+1}} \left(Q \frac{4n+2}{nh \dfrac{W_1}{H_1}}\right)^n$$

or

$$dP = +\eta \cot\theta \left(Q \frac{4n+2}{n} \frac{H_1}{W_1}\right)^n h^{-(3n+1)} dh \tag{12}$$

Integrating between the limits H_1 and H_2, we obtain the total pressure drop ΔP, as before

$$\Delta P = -\eta \cot\theta \left(Q \frac{4n+2}{n} \frac{H_1}{W_1}\right)^n \int_{H_1}^{H_2} h^{-(3n+1)}\, dh \tag{13}$$

which finally gives

$$\Delta P = \frac{\eta \cot\theta}{3n} \left(Q \frac{4n+2}{n} \frac{H_1}{W_1}\right)^n (H_2{}^{-3n} - H_1{}^{-3n}) \tag{14}$$

This equation is exact and not restricted to the limitations of eq. (11).

The treatment leading to eqs. (11) and (14) removes the justified criticism, on dimensional grounds, of the earlier treatment (loc. cit.). Furthermore, eq. (14) is of precisely the same form as that obtained for tapering (truncated cone) dies with a constant (circular) cross-sectional shape, which (loc. cit.) remains

$$\Delta P = \frac{2\eta \cot\theta}{3n} \left(Q \frac{3n+1}{\pi n}\right)^n (R_2{}^{-3n} - R_1{}^{-3n}) \tag{15}$$

Inspection of eqs. (14) and (15) shows that the length L of the die does not appear in them; neither does W_2 appear in eq. (14). These are, of course, implicit in that they are defined, respectively, by H_1, H_2, W_1, and θ in the constant-shape double-taper wide-slit die and by R_1, R_2, and θ in the truncated cone die.

It is also worth looking at the case of a die shaped as a truncated square pyramid. The following relationships hold for a square *prism* ($w = h = a$; length $= L$):

$$\tau = \frac{\Delta P\, a}{4L}$$

$$\dot\gamma_N = \frac{6Q}{a^3}$$

$$\dot\gamma = \frac{4n+2}{n} \frac{Q}{a^3} \therefore$$

$$\Delta P = 4\eta L \left(Q \frac{4n+2}{n}\right)^n a^{-(3n+1)}.$$

In the case of the *square truncated pyramid*, the incremental pressure drop dP is

$$dP = 4\eta\, dl \left(Q \frac{4n+2}{n}\right)^n a^{-(3n+1)}$$

and since, by trigonometry,

$$dl = -\frac{da \cot\theta}{2}$$

$$dP = -2\eta \cot\theta \left(Q \frac{4n+2}{n}\right)^n a^{-(3n+1)}\, da$$

and integrating,

$$\Delta P = -2\eta \cot\theta \left(Q \frac{4n+2}{n}\right)^n \int_{a_1}^{a_2} a^{-(3n+1)}\, da$$

or finally

$$\Delta P = \frac{2\eta \cot\theta}{3n} \left(Q \frac{4n+2}{n}\right)^n ({a_2}^{-3n} - {a_1}^{-3n}) \quad (16)$$

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